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EFFECT OF HINGE-MOMENT PARAMETERS ON ELEVATOR *Loading*

STICK FORCES IN RAPID MANEUVERS

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ADVANCE RESTRICTED REPORT

EFFECT OF HINGE-MOMENT PARAMETERS ON ELEVATOR  
STICK FORCES IN RAPID MANEUVERS

By Robert T. Jones and Harry Greenberg

SUMMARY

The importance of the stick force per unit normal acceleration as a criterion of longitudinal stability and the critical dependence of this gradient on elevator hinge-moment parameters have been shown in previous reports. The present report continues the investigation with special reference to transient effects for maneuvers of short duration.

The analysis made showed that different combinations of elevator parameters which give the same stick force per unit acceleration in turns give widely different force variations during the entries into and recoveries from steady turns and during maneuvers of short duration such as abrupt pull-ups. A combination of relatively large negative values of the restoring tendency  $C_{h\delta}$  and the floating tendency  $C_{h\dot{\alpha}}$ , approaching those of an unbalanced elevator, results in a stick force that is high during the initial stage of a pull-up and then decreases, and may even reverse, as the acceleration is reduced at the end of the maneuver. The stick force per unit acceleration is greater for abrupt than for gradual control movements.

If the negative value of  $C_{h\delta}$  is reduced so that the corresponding value of  $C_{h\dot{\alpha}}$  becomes slightly positive, the reversal of force may be eliminated and the force may be brought nearly in phase with the acceleration. There is a limit to the permissible reduction of the value of  $C_{h\delta}$ , however, because as  $C_{h\delta}$  approaches zero the stick force per unit acceleration may become lower for abrupt than for gradual maneuvers and may thus lead to undesirably low stick forces at the beginning of the maneuver.

## INTRODUCTION

The stick force per unit normal acceleration as measured in steady turns or pull-outs, which was proposed as a criterion of longitudinal handling in reference 1, is now generally accepted as a basic measure of longitudinal stability. The critical dependence of this stick-force gradient on elevator hinge-moment parameters and on mass unbalance of the control system was shown in reference 2. It was found that a given stick-force gradient can be obtained by any of a series of combinations of these parameters satisfying certain prescribed relations.

Further consideration of the problem and some recent flight experience, however, have shown the need for investigating the transient effects that occur during the change from steady unaccelerated flight to steady accelerated flight. These transient effects cause a difference between the stick-force gradients in a steady turn and in a maneuver of short duration such as a pull-up.

The purpose of the present report is to investigate the variation of elevator stick force and normal acceleration during the transition interval preceding the steady turn and also during turns or pull-ups of short duration. The effect of combinations of hinge-moment parameters is considered; each combination is chosen to give the same stick-force gradient in a steady maneuver. Time histories of the stick force and normal acceleration are found for predetermined variations of elevator deflection. An attempt is made to explain and to suggest a remedy for the large variations of stick force with time observed during pull-ups of short duration on different airplanes in flight. A previous analysis, somewhat similar to the present one, was made in England (reference 3) but included a smaller range of hinge-moment parameters.

## SYMBOLS

- A aspect ratio of wing
- b wing span

|                   |   |
|-------------------|---|
| $C_h$             | elevator hinge-moment coefficient $\left( \frac{H}{qS_e c_e} \right)$   |
| $C_L$             | airplane lift coefficient $\left( \frac{\text{Lift}}{qS} \right)$   |
| $C_m$             | pitching-moment coefficient about airplane center of gravity $\left( \frac{\text{Pitching moment}}{qSc} \right)$                |
| $c$               | wing chord  |
| $c_e$             | elevator chord  |
| $D$               | differential operator $(d/ds)$  |
| $F_s$             | stick force, pounds   |
| $F_1, \dots, F_5$ | cases representing particular combinations of hinge-moment parameters   |
| $F_n$             | stick-force gradient in maneuvers $\left( \frac{dF_s}{dn} \right)$  |
| $g$               | acceleration of gravity   |
| $H$               | hinge moment; positive when tends to lower elevator   |
| $H_o$             | mass moment of elevator control system about elevator hinge; positive when tends to lower elevator                              |
| $h$               | $h = \frac{lH_o}{\rho S_e c_e c}$   |
| $k_Y$             | radius of gyration of airplane about Y-axis   |
| $l_h$             | tail length, half-chords  |
| $m$               | mass of airplane  |
| $n$               | normal acceleration per $g$ of airplane due to curvature of flight path; accelerometer reading minus component of gravity force |
| $q$               | dynamic pressure  |
| $S$               | wing area   |

|              |   |
|--------------|---|
| $S_e$        | elevator area   |
| $s$          | distance traveled, half-chords $(2Vt/c)$  |
| $T$          | period of elevator motion   |
| $t$          | time  |
| $u$          | independent variable used in Duhamel's integral                                 |
| $V$          | velocity  |
| $x_{a.c.}$   | distance between center of gravity and aerodynamic center; positive when stable |
| $d\delta/dx$ | deflection of elevator per unit movement of stick, radians per foot             |
| $\alpha$     | angle of attack, radians  |
| $\alpha_t$   | angle of attack at tail, radians  |
| $\delta$     | deflection of elevator; positive downward                                       |
| $\theta$     | angle of pitch of airplane  |
| $\lambda$    | root of stability equation  |
| $\mu$        | airplane-density parameter $(m/\rho S b)$                                       |
| $\rho$       | mass density of air   |

Subscript:

max      maximum

Subscripts  $\alpha$ ,  $D\alpha$ ,  $D^2\alpha$ ,  $\alpha_t$ ,  $D\theta$ ,  $\delta$ , and  $D\delta$  indicate derivatives; for example,  $C_{mD\theta} = \frac{\partial C_m}{\partial D\theta}$ . A dot over a symbol indicates differentiation with respect to time.

#### METHOD OF ANALYSIS

The following assumptions are made in the present analysis:

- (1) Variation in forward speed is negligible
- (2) Stability derivatives are constant; that is, any possible nonlinearity of coefficients is negligible
- (3) Effects of power are negligible
- (4) Effects of control-system moment of inertia are negligible
- (5) Control-system mass unbalance is all located at airplane center of gravity

The equations of motion of an airplane subjected to a prescribed elevator motion can be obtained from reference 2. If forward speed is assumed constant, there are three equations of motion. The first two equations determine the motion of the airplane if the control motion is specified. The third equation determines the hinge-moment coefficient, which depends on the motion of the control surface and the airplane. These equations are

$$\left( \frac{C_{L\alpha}}{2} + 2A_{\mu}D \right) \alpha - 2A_{\mu}D\theta = 0 \quad (1)$$

$$(C_{m\alpha} + C_{mD\alpha}D + C_{mD^2\alpha}D^2)\alpha + (C_{mD\theta} - 2A_{\mu}k_Y^2D)\theta = -C_{m\delta}\delta \quad (2)$$

$$[C_{h\alpha} + (C_{hD\alpha}D - h)D + C_{hD^2\alpha}D^2]\alpha + (C_{hD\theta} + h)D\theta + (C_{h\delta} + C_{hD\delta}D)\delta = C_h \quad (3)$$

Equations (1) and (2) are used to solve for  $\alpha$  in terms of  $\delta$ . The solution can be expressed in determinant form as

$$\frac{\alpha}{\delta} = \frac{-2A_{\mu}C_{m\delta}}{\begin{vmatrix} \frac{C_{L\alpha}}{2} + 2A_{\mu}D & -2A_{\mu} \\ C_{m\alpha} + C_{mD\alpha}D + C_{mD^2\alpha}D^2 & C_{mD\theta} - 2A_{\mu}k_Y^2D \end{vmatrix}} \quad (4)$$

If  $\delta$  is given as a function of time, the solution for  $\alpha$  is found by the method of operational calculus as follows: First  $\alpha$  is found for a unit change in  $\delta$ . This solution is obtained from

$$\alpha = \frac{-2A\mu C_{m\delta}}{F(D)} = -2A\mu C_{m\delta} \left[ \sum \frac{e^{\lambda s}}{\lambda F'(\lambda)} + \frac{1}{F(0)} \right] \quad (5)$$

where  $F(D)$  is the determinant given in equation (4) and  $\lambda$  represents the roots of  $F(D) = 0$ . The solution for  $\alpha$  (equation (5)) may be denoted by  $\bar{\alpha}(s)$ . The value of  $\alpha$  for a given variation of  $\delta$  is then given by Duhamel's integral, which is

$$\alpha = \bar{\alpha}(s) \delta(0) + \int_0^s \bar{\alpha}(s-u) \delta'(u) du$$

By a similar procedure  $D\theta$  can be found for a prescribed variation of  $\delta$ . The angle of attack at the tail can then be found from

$$\alpha_t = \frac{\partial \alpha_t}{\partial \alpha} \alpha + l_h D\theta$$

The normal acceleration, which is considered positive upward, is proportional to the change in angle of attack  $\alpha$  and is given by

$$n = \frac{v^2}{cg} \frac{C_{L\alpha}}{2A\mu} \alpha$$

The value of the stick force can be obtained by substituting the derived values of  $\alpha$  and  $D\theta$  and the given value of  $\delta$  in the hinge-moment equation (equation (3)). The relation between the stick force and  $C_h$  is simply

$$F_s = \frac{1}{2} \rho v^2 S_e c_e C_h \frac{d\delta}{dx}$$

The assumed variation of elevator deflection with time is illustrated in figure 1 and can be represented analytically by

$$\delta = \delta_{\max} \left( \frac{1}{2} - \frac{1}{2} \cos \frac{2\pi}{T} t \right)$$

The calculations were made for a pursuit airplane for five different combinations of the hinge-moment parameters  $C_{h_{a_t}}$ ,  $C_{h_{\delta}}$ , and  $h$ ; for three different durations of the maneuver  $T$ ; and for three different center-of-gravity locations. These five different combinations of the hinge-moment parameters were selected to give, for one center-of-gravity location, the same stick-force gradient in a steady turn, as determined by the formula for stick-force gradient in a gradual pull-up or steady turn given in reference 2, which is

$$F_n = \frac{\rho S_e c_{e cg}}{4} \frac{d\delta}{dx} \left( \frac{4A\mu C_{h_a}}{C_{L_a}} + C_{h_{D\theta}} - \frac{4A\mu C_{h_{\delta}} C_{m_a}}{C_{L_a} C_{m_{\delta}}} - \frac{C_{h_{\delta}} C_{m_{D\theta}}}{C_{m_{\delta}}} + h \right)$$

The locus of points in the  $C_{h_{a_t}} C_{h_{\delta}}$  plane corresponding to a value of the stick-force gradient of 5 pounds per g and a center-of-gravity location  $7\frac{1}{2}$  percent chord ahead of the aerodynamic center is shown in figure 2 for a mass-balanced and also for a mass-unbalanced elevator. The amount of unbalance corresponding to the line marked  $h = 5$  would require a pull of 15 pounds on the control stick for balance. The five points marked  $F_1, \dots, F_5$  represent the combinations of the hinge-moment parameters used in the calculations.

#### NUMERICAL VALUES USED IN ANALYSIS

The following parameters were used in the analysis:

|                   |                            |
|-------------------|----------------------------|
| $C_{L_a}$         | 4.3                        |
| $\mu$             | 12.5                       |
| $A$               | 6                          |
| $C_{m_a}$         | -0.348, -0.195, or -0.0464 |
| $x_{a.c.}$        | 0.075c, 0.042c, or 0.01c   |
| $C_{m_{D_a}}$     | -8.9                       |
| $C_{m_{D^2_a}}$   | 23.2                       |
| $C_{m_{D\theta}}$ | -15.3                      |



|   |                       |
|---|-----------------------|
| $k_Y$ , half-chords . . . . .   | 1.5                   |
| $C_{m\delta}$ . . . . .   | -1.54                 |
| $l_h$ , half-chords . . . . .   | 6.6                   |
| $d\delta/dz$ , radian of elevator motion per foot of stick travel . . . . . | 0.5                   |
| $C_{h\alpha}$ . . . . .   | $0.514C_{h\alpha t}$  |
| $C_{hD\alpha}$ . . . . .  | $3.22C_{h\alpha t}$   |
| $C_{hD^2\alpha}$ . . . . .  | $-10.55C_{h\alpha t}$ |
| $C_{hD\delta}$ . . . . .  | -1                    |

The following dimensions and density were assumed:

|   |         |
|---|---------|
| $c$ , feet . . . . .                                    | 7       |
| $c_e$ , feet . . . . .                                  | 2       |
| $S_e$ , square feet . . . . .                           | 30      |
| $\rho$ , slug/cu ft; at altitude of 10,000 feet . . . . | 0.00176 |

The foregoing airplane derivatives are for an airplane having a wing loading of 30 pounds per square foot. Five combinations of hinge-moment parameters selected to give a stick-force gradient of 5 pounds per g in a steady pull-up when the center-of-gravity location is  $7\frac{1}{2}$  percent chord ahead of the aerodynamic center (see fig. 2) are as follows:

| Case  | $C_{h\alpha t}$ | $C_{h\delta}$ | $h$  |
|-------|-----------------|---------------|------|
| $F_1$ | -0.1            | -0.230        | 0    |
| $F_2$ | 0               | -.065         | 0    |
| $F_3$ | .039            | 0             | 0    |
| $F_4$ | -.1             | -.035         | 5    |
| $F_5$ | 0               | 0             | 1.65 |

All these values were used in calculating the variation in stick force during a maneuver for  $x_{a.c.} = 0.075c$ . For qualitative comparison, case  $F_1$  may be taken to represent a normal elevator with a fairly high trailing

tendency and a moderate amount of blunt-nose inset-hinge balance. The characteristics of  $F_2$  or  $F_3$  could be achieved by the use of a sharp-nose inset-hinge balance, a horn balance, or a beveled trailing edge;  $F_4$  combines a large amount of inset-hinge balance with a bobweight at the control stick;  $F_5$  is the case in which the stick force is due entirely to the bobweight. Two more-rearward center-of-gravity locations ( $x_{a.c.} = 0.042c$  and  $0.01c$ ) were also assumed, and the stick force in maneuvers was worked out for cases  $F_1$ ,  $F_3$ , and  $F_5$ .

## RESULTS

Curves of stick force and normal acceleration for a varying elevator deflection are shown in figures 3, 4, and 5 for  $T = 4, 2$ , and  $1$  seconds, respectively, for  $V = 400$  miles per hour, and for  $x_{a.c.} = 0.075c$ . In these curves, the stick force for  $F_1$  reaches a maximum value before the peak acceleration and reverses direction in the latter part of the cycle. This effect becomes more pronounced as the duration of the maneuver becomes shorter. The curves for  $F_2$ ,  $F_3$ ,  $F_4$ , and  $F_5$  show a progressively smaller phase difference between the stick force and the acceleration. The stick-force curve for  $F_4$  is most nearly in phase with the acceleration curve.

The effect of center-of-gravity location on the stick-force gradient in steady turns or pull-ups can be shown in diagrams of the type of figure 2. Figure 6, for example, shows that the "maneuver point" (c.g. location for zero stick force per g) for case  $F_1$  is  $4.2$  percent chord ahead of the aerodynamic center (point where  $C_{m_\alpha} = 0$ ). For center-of-gravity locations behind the maneuver point, the stick-force gradient for case  $F_1$  is negative. The stick forces for  $F_3$  and  $F_5$ , however, are unaffected by center-of-gravity location.

The time histories of the stick forces in a 2-second maneuver for the cases shown in figure 6 for  $x_{a.c.} = 0.042c$  and  $0.01c$  are plotted in figures 7 and 8. In figure 7, the stick force corresponding to  $F_1$  (c.g. at maneuver point)

is positive at first and then reverses and becomes negative. The maximum values of the positive and negative forces are approximately equal. As the center of gravity is moved behind the maneuver point for  $F_1$  (fig. 8), the negative maximum force is greater than the positive; this increase would be expected since a negative force is required to hold the airplane in a steady turn. The stick forces for  $F_3$  and  $F_5$  remain positive. The elevator deflection required to produce a given acceleration, however, decreases as the center of gravity moves rearward.

Airplane speed has no effect on the shape of the stick-force and acceleration curves, if compressibility effects are neglected and if the product of speed and duration of maneuver is held constant; for example, the shape of the curves of figures 3 to 5 is unchanged if the speed is halved and the duration is doubled. The effect of increasing speed therefore is the same as the effect of increasing duration in the same ratio.

## DISCUSSION

Before the various elevator cases and degrees of stability for which the computations were made are discussed, it appears desirable to explain the effects of the separate parameters that combine to give the resultant elevator forces in pull-ups. These effects, as already stated, are the variation of hinge-moment coefficient with elevator deflection, as indicated by  $C_{h\delta}$ ; the variation of hinge-moment coefficient with angle of attack at the tail, as indicated by  $C_{h\alpha_t}$ ; the variation of hinge moment with angular velocity of the elevator about its hinge; the mass unbalance (bobweight effect); and the effective moment of inertia of the elevator system.

Because preliminary computations indicated that the inertia of the elevator system had a negligible effect on the stick force for the shortest maneuver assumed, it was neglected in the analysis. For airplanes larger than the one considered in this report and for other special cases, inertia of the elevator system may be an important factor.

The influence of the important parameters is shown in figure 9, which gives a breakdown of the factors

contributing to the stick-force curve for case  $F_4$  in figure 5. Case  $F_4$  was chosen because it was the only condition in which all the parameters were combined.

Figure 9 shows that the effect of  $C_{h\delta}$  is to produce a component of stick force in phase with elevator deflection. The magnitude of this component of the stick force depends solely on the elevator deflection at a given speed and is independent of the duration of the maneuver.

The normal acceleration produced by the elevator decreases as the duration of the maneuver is made shorter. The stick force per unit acceleration due to the  $C_{h\delta}$  term therefore increases as the maneuver becomes more rapid.

The effect of the mass unbalance of a bobweight is to contribute a component of force that is in phase with and solely dependent on the normal acceleration of the airplane. The stick-force gradient due to the bobweight is therefore independent of duration of maneuver. Although figure 9 deals with a mass unbalance that tends to depress the trailing edge of the elevator, in the general case the unbalance may be of the opposite sign so that push instead of pull forces result.

The effect of  $C_{h\dot{\alpha}}$  is similar to that of the bobweight since the component of force caused by  $C_{h\dot{\alpha}}$  is nearly in phase with the acceleration. The slight difference in phase between the values of  $\dot{\alpha}$  and  $n$  is the effect of the rate of change of airplane angle of attack. For maneuvers of short duration, this slight phase shift causes a noticeable difference between the action of  $C_{h\dot{\alpha}}$  and of a bobweight.

The component of force due to the angular velocity of the elevator may be very important for maneuvers of short duration. It has the effect of reducing the stick-force gradients in cases in which the maximum force occurs after the elevator has reached maximum deflection.

The cases for which the results are presented in figures 3 to 5 were chosen to show the effects of different combinations of the hinge-moment parameters

subject to the designer's control. The parameter  $C_{hD\delta}$  is the same for all cases. In case  $F_1$ , the desired stick force for a steady turn is achieved by a balance of relatively large negative values of  $C_{h\delta}$  and  $C_{h_{at}}$ .

The stick forces due to these two parameters are in opposite directions so that the net value in a steady turn is due to the difference in their effects. In a maneuver of the type shown in figure 1, the elevator-deflection curve leads the normal-acceleration curve; hence  $C_{h\delta}$  has the predominating effect in the initial stages of the maneuver and the negative  $C_{h_{at}}$ , in the later stages. This fact accounts for the high stick forces in the first half of the maneuver and the reversal of force in the second half for case  $F_1$ . The difference is more noticeable in the shorter maneuvers. As the duration of the maneuver decreases, the lag between airplane motion and elevator deflection becomes greater and the maximum value of the acceleration for the given elevator deflection becomes smaller. Both of these factors tend to reduce the importance of the  $C_{h_{at}}$  component in the early part of the maneuver and to increase the maximum force required for a given maximum acceleration. This variation of maximum force per unit maximum acceleration shown in figure 10 is quite large.

For case  $F_2$ , the desired stick force for steady turns is achieved through the action of  $C_{h\delta}$  alone. All curves for  $F_2$  would have the same magnitude for any duration of maneuver and would be in phase with the elevator-deflection curve but for the contribution of  $C_{hD\delta}$ . The effect of  $C_{hD\delta}$  increases with the rapidity of the elevator movement and causes a phase shift in the force curve relative to the elevator deflection, which results in a slight increase in the maximum value for the shortest maneuver. A slight push force near the end of the maneuver is produced by  $C_{hD\delta}$ . Figure 10 shows that in case  $F_2$  the maximum force per unit maximum acceleration increases as the maneuver is shortened although not so much as in case  $F_1$ .

The balance is achieved in case  $F_3$  through action of  $C_{h_{at}}$  alone. In this case, the maximum stick force

attributed to  $C_{h_{at}}$  is nearly in phase with the acceleration and, consequently, the maximum value occurs after maximum elevator deflection when the elevator is being moved back to its original position. The forces at the beginning of the maneuver are consequently smaller than in cases  $F_1$  and  $F_2$  and may be too small for satisfactory handling qualities. The effect of  $C_{h_{D\delta}}$  is to decrease the maximum force by an increasing amount as the maneuver becomes shorter. The discontinuity in the  $F_3$  curve (and also in the  $F_4$  and  $F_5$  curves) for the 1-second maneuver results from the disappearance of the  $C_{h_{D\delta}}$  component at the completion of the elevator motion. Figure 10 shows that the maximum force per unit maximum acceleration for case  $F_3$  decreases as the maneuver is shortened; this effect is primarily a result of the action of  $C_{h_{D\delta}}$ .

For case  $F_4$ , the stick force for steady turns is achieved mainly by a balance of negative  $C_{h_{at}}$  and bobweight effects. As a result of the large mass unbalance required, the maximum force in the 1-second maneuver occurs at the end of the elevator motion.

The stick force is achieved solely through the action of mass unbalance, or a bobweight, in case  $F_5$ . Computations have been made for only the 1-second maneuver. The action of the bobweight, as previously mentioned, is similar to that of  $C_{h_{at}}$  but for a slight phase shift. The phase shift for a maneuver of short duration is sufficient to reduce the adverse influence of  $C_{h_{D\delta}}$ . This case would show a slightly greater decrease of maximum force per unit maximum acceleration than case  $F_3$  with decreased duration of the maneuver.

The change of stick force with center-of-gravity location for case  $F_1$ , shown in figures 7 and 8, is caused by the greater angular response of the airplane to a given elevator deflection that occurs with reduced stability. The greater response changes the balance between the  $C_{h_{at}}$  and  $C_{h_{\delta}}$  components. If the stick

force is independent of  $C_{h\delta}$ , as in cases  $F_3$  and  $F_5$ , the form of the stick-force curves is unchanged by variation of the center-of-gravity location. Figure 11 shows that the variation of maximum force per unit maximum acceleration in a rapid maneuver with center-of-gravity location becomes less as the value of  $C_{h\delta}$  is reduced.

The adjustment of the elevator parameters so that the stick forces for steady turns are directly proportional to the normal acceleration produced and independent of center-of-gravity location is generally conceded to be desirable. It appears possible from the analysis to accomplish these conditions by making the stick forces depend primarily on  $C_{h_{at}}$  or on a bobweight, provided the

entrance and recovery are made slowly. It is not definitely known whether this condition of strict proportionality is desired in maneuvers of short duration. In these cases, however, when the entry and recovery are of necessity rapid, strict proportionality between stick force and acceleration appears impossible because of the action of  $C_{h_{D\delta}}$ . According to figure 10, a stick-force

gradient that is independent of duration of maneuver but varies somewhat with center-of-gravity location can be obtained for a case intermediate between  $F_2$  and  $F_3$ . This case would correspond to a certain amount of negative  $C_{h\delta}$  and positive  $C_{h_{at}}$  and would also result in

higher stick forces at the start of the maneuver. A bobweight that increases the stick forces can be substituted for the positive  $C_{h_{at}}$ .

#### CONCLUDING REMARKS

A small stick-force gradient in steady turns can be obtained with fairly large negative values of the restoring tendency  $C_{h\delta}$  and the floating tendency  $C_{h_{at}}$ , approaching those of an unbalanced elevator. Although suitable for slow maneuvers, this combination of parameters leads to a high initial value followed by a reversal of the stick force in abrupt maneuvers. This difficulty can be avoided and the stick force can be made to follow

closely in phase with the airplane normal acceleration during both abrupt and slow maneuvers by decreasing the value of  $C_{h\delta}$  and by making  $C_{h\alpha_t}$  slightly positive.

If  $C_{h\delta}$  is made zero, the stick-force gradient depends entirely on a positive value of  $C_{h\alpha_t}$  and is unaffected by the location of the airplane center of gravity. In this condition, however, the stick force required to initiate a maneuver may be undesirably light. In order to prevent undesirably light stick forces at the beginning of a maneuver, a small negative  $C_{h\delta}$  must be retained.

The use of a bobweight in the elevator control system has an effect similar to that of increasing  $C_{h\alpha_t}$  although, in rapid maneuvers, there are slight phase differences in the stick-force variations.

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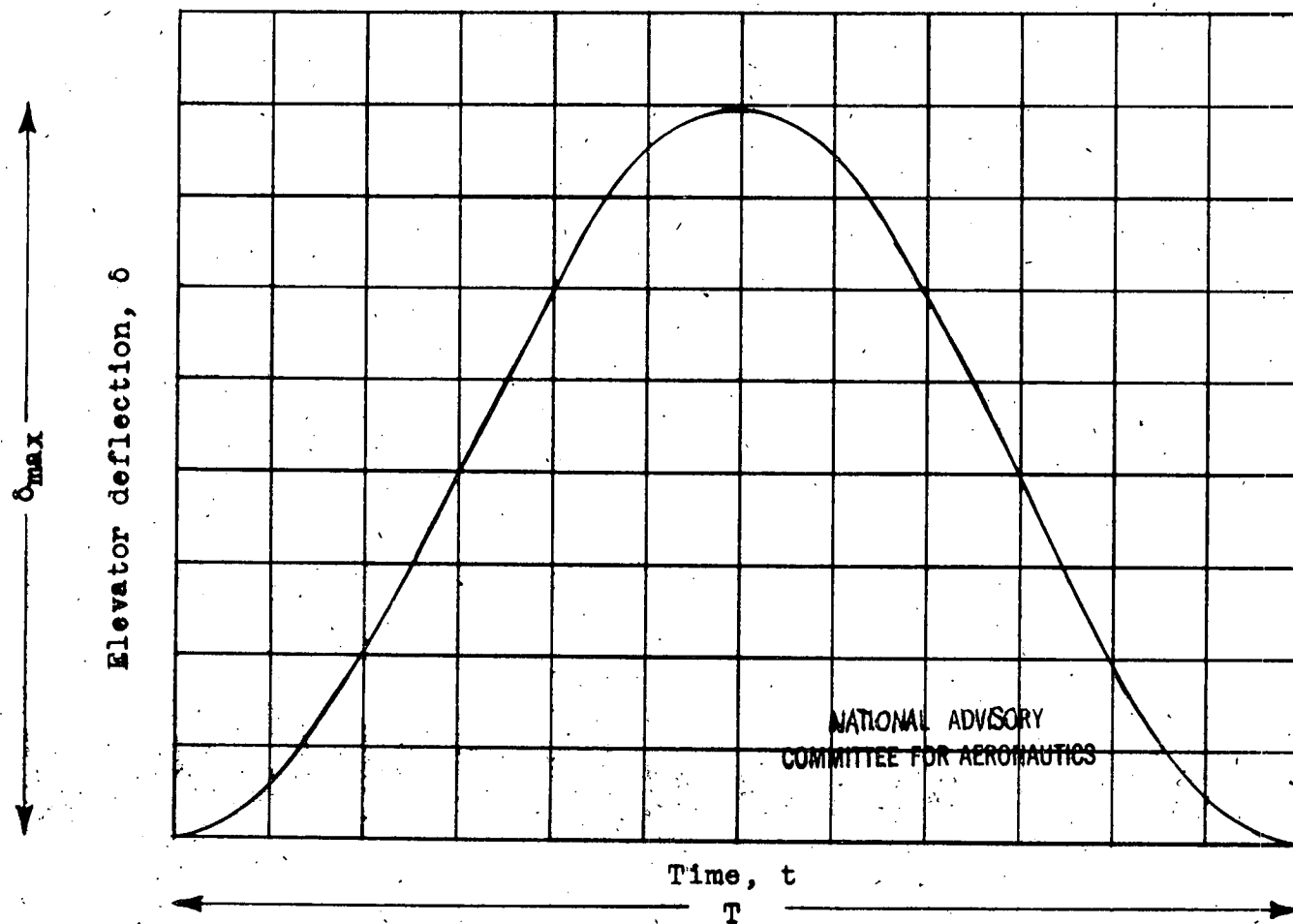


Figure 1.- Shape of curve of elevator deflection against time assumed in the analysis.

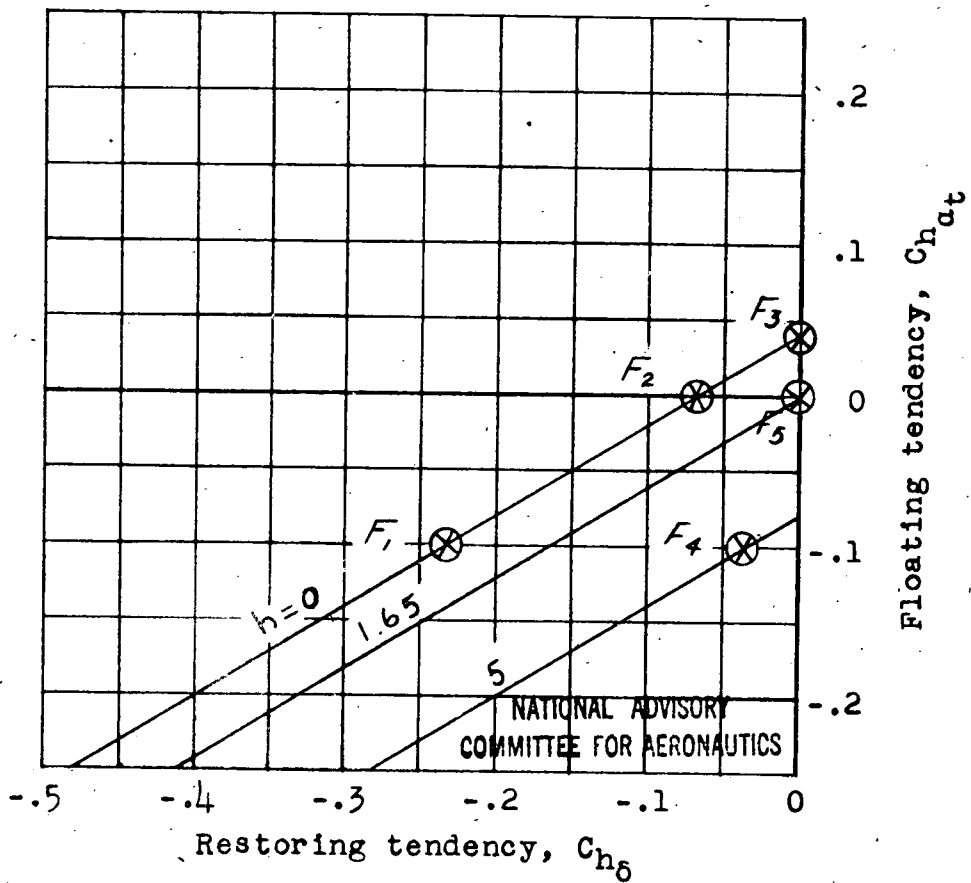


Figure 2.- Lines of constant stick-force gradient showing combinations of hinge-moment parameters used.  $F_n = 5$  pounds per g;  $x_{a.c.} = 0.075c$ .

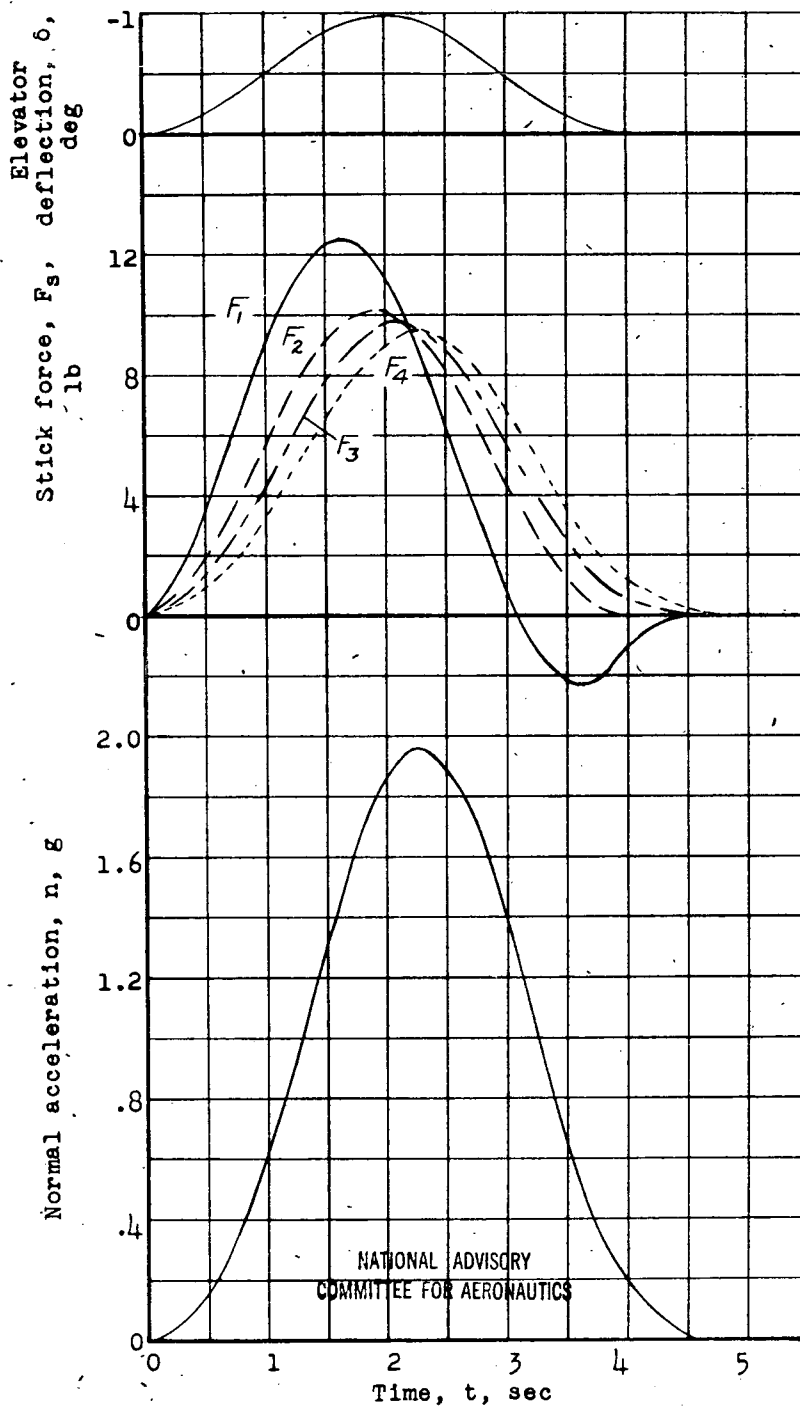


Figure 3.- Stick force and normal acceleration due to rapid elevator motion.  $T = 4$  seconds;  $V = 400$  miles per hour  $x_{a.c.} = 0.075c$ .

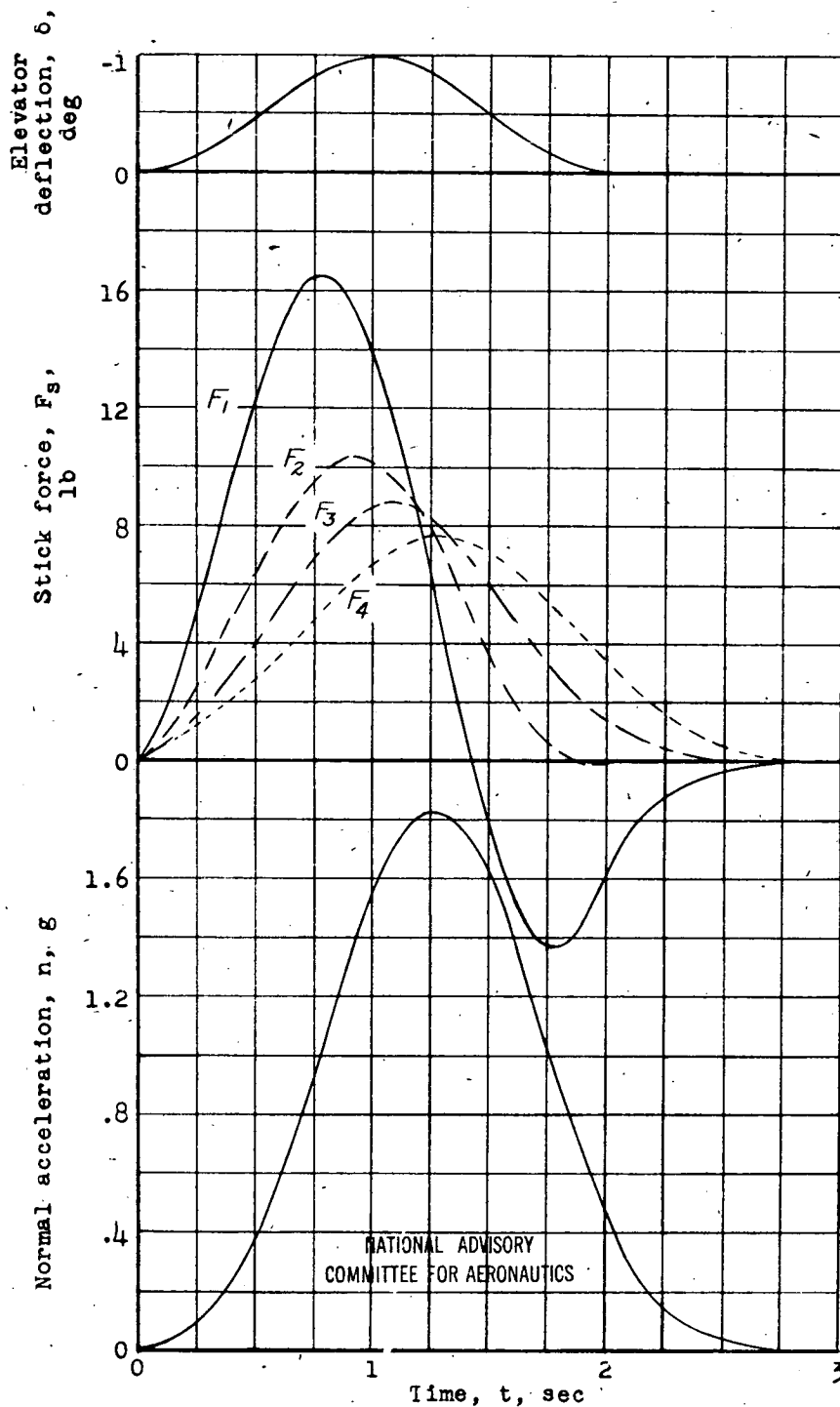


Figure 4.- Stick force and normal acceleration due to rapid elevator motion.  $T = 2$  seconds;  $V = 400$  miles per hour;  $x_{a.c.} = 0.075c$ .

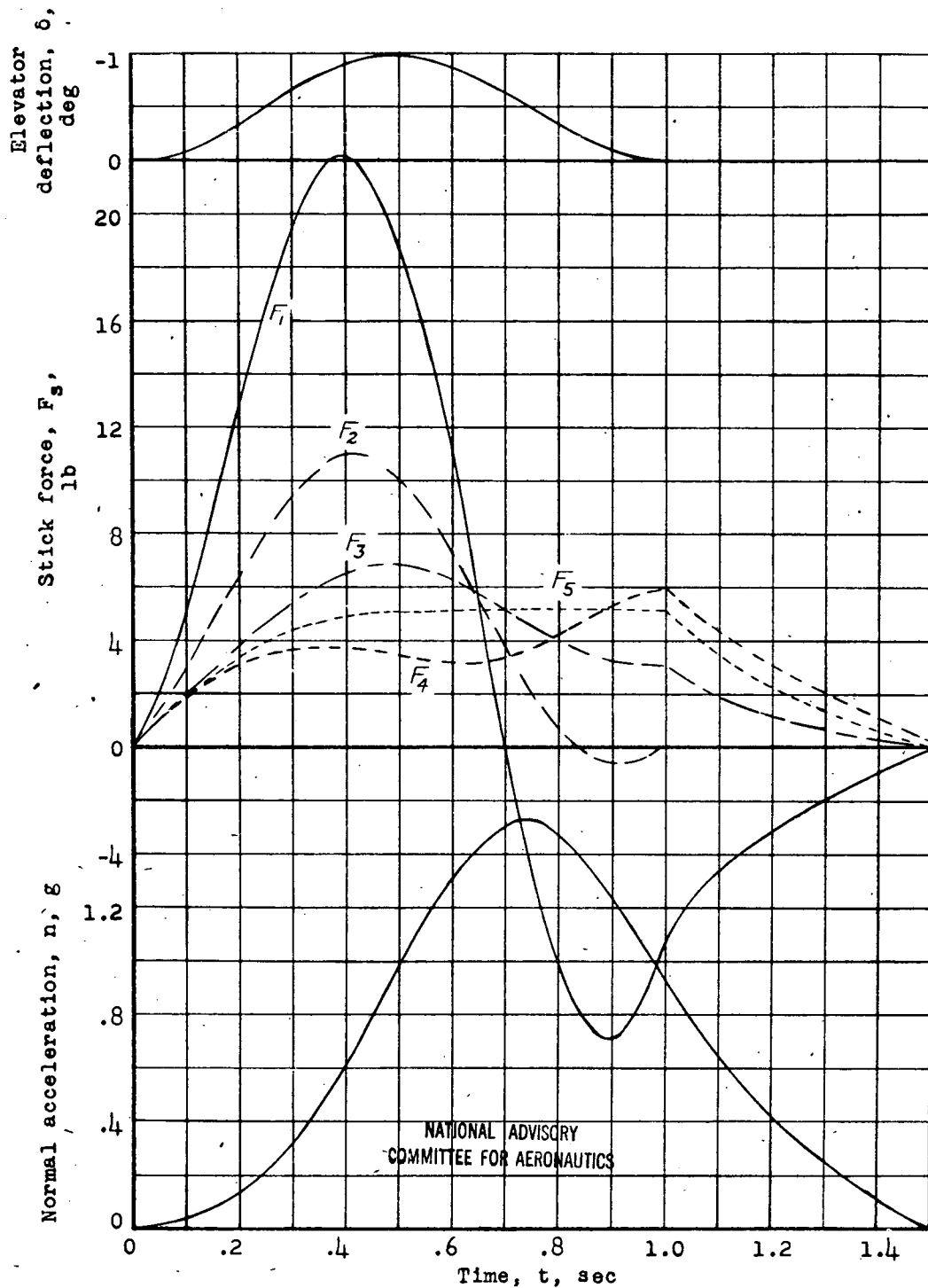


Figure 5.- Stick force and normal acceleration due to rapid elevator motion.  $T = 1$  second;  $V = 400$  miles per hour;  $x_{a.c.} = 0.075c$ .

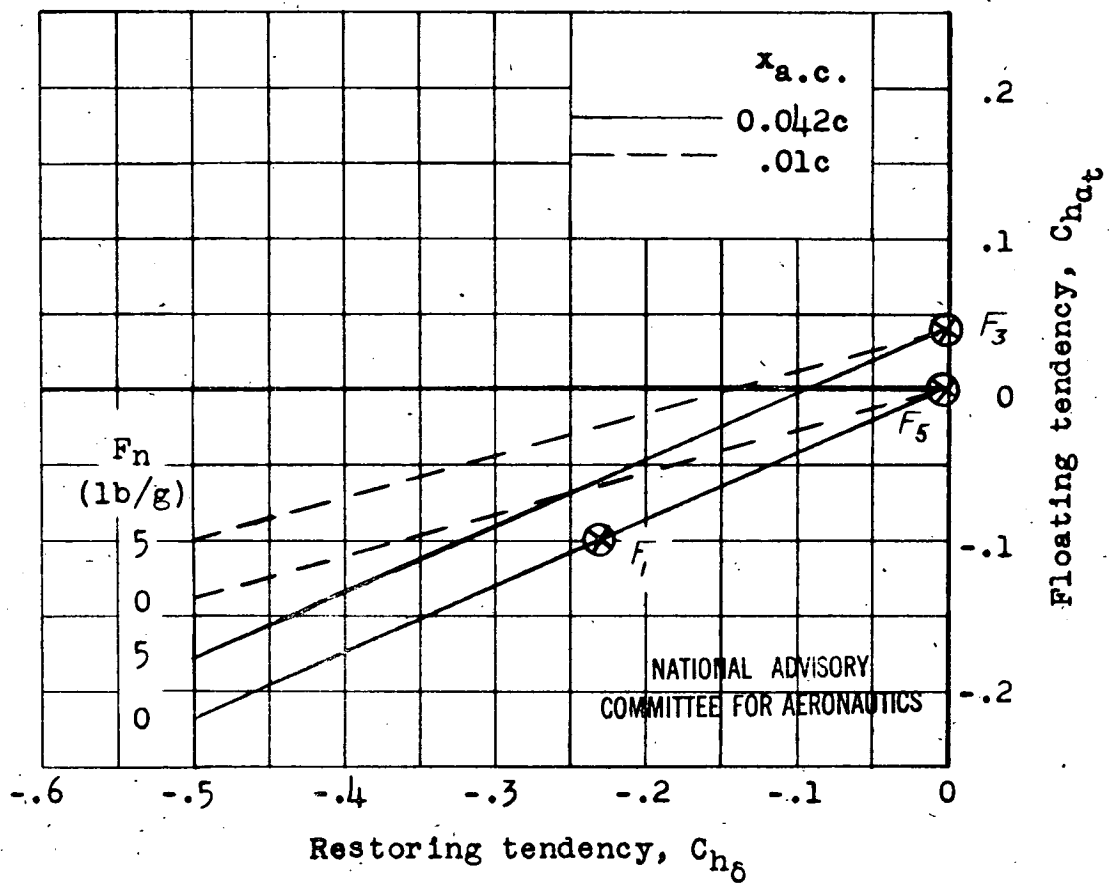


Figure 6.- Lines of constant stick-force gradient.  
 $F_n = 0$  and 5 pounds per g.

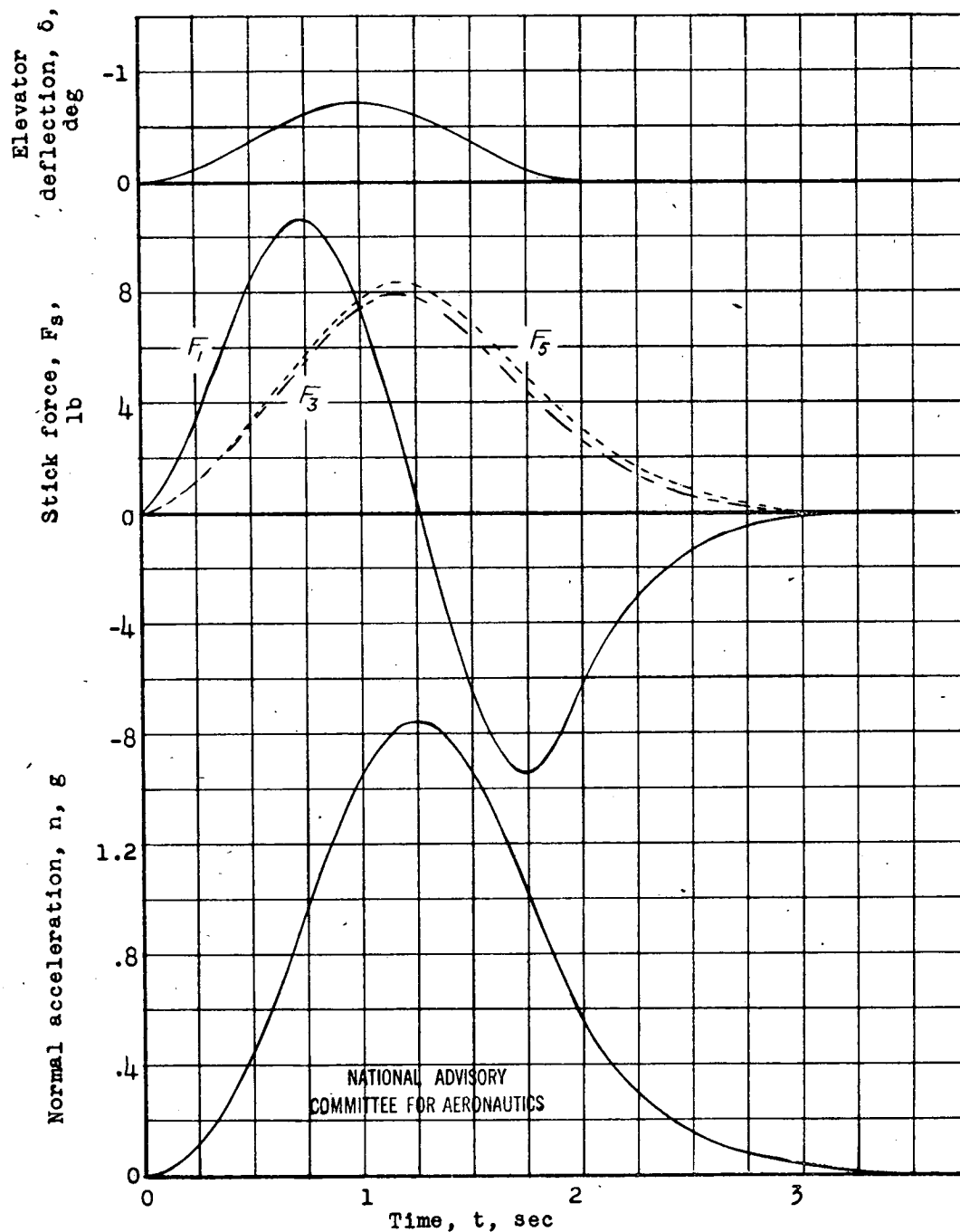


Figure 7.- Stick force and normal acceleration due to rapid elevator motion.  $T = 2$  seconds;  $V = 400$  miles per hour;  $x_{a.c.} = 0.042c$ .



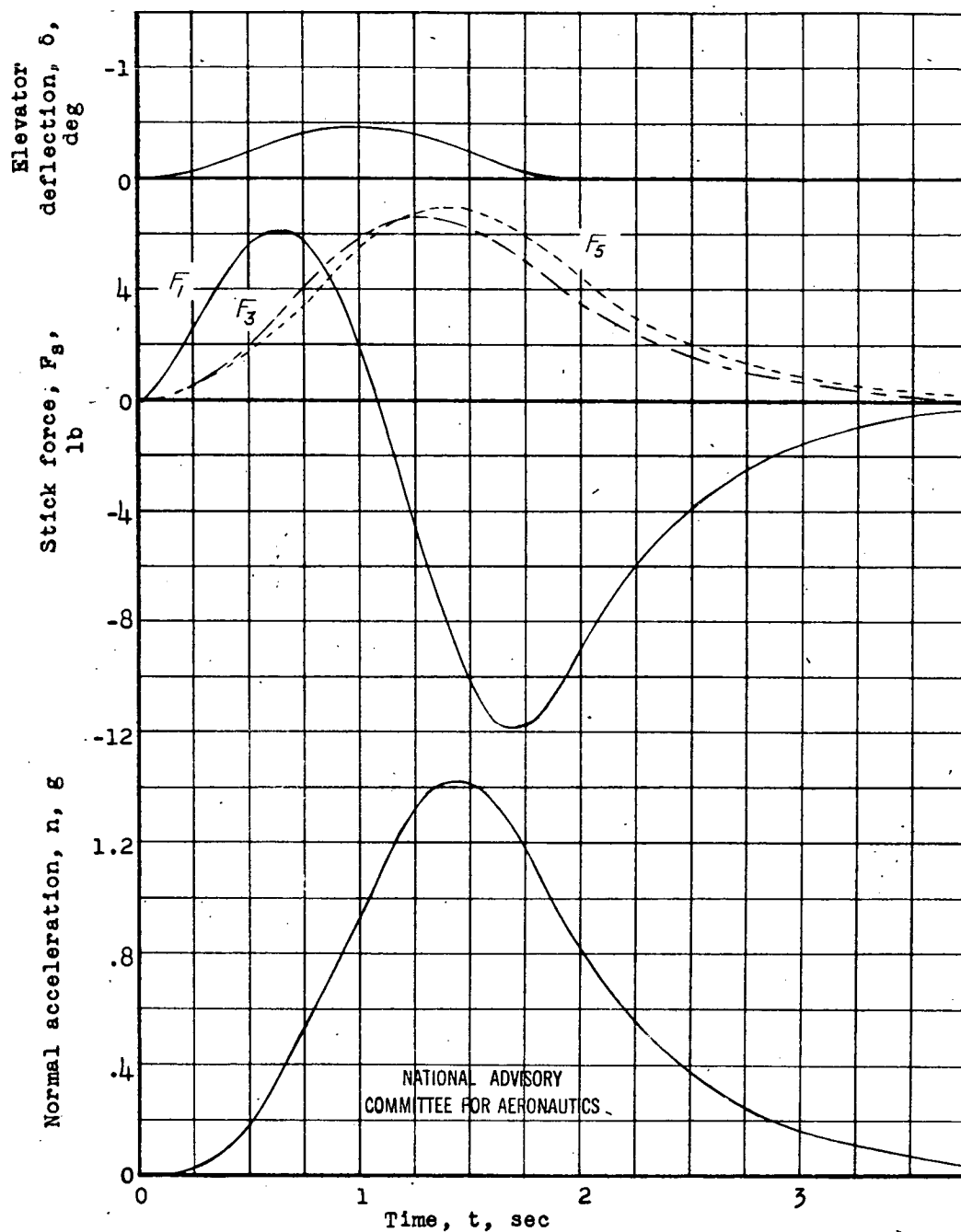
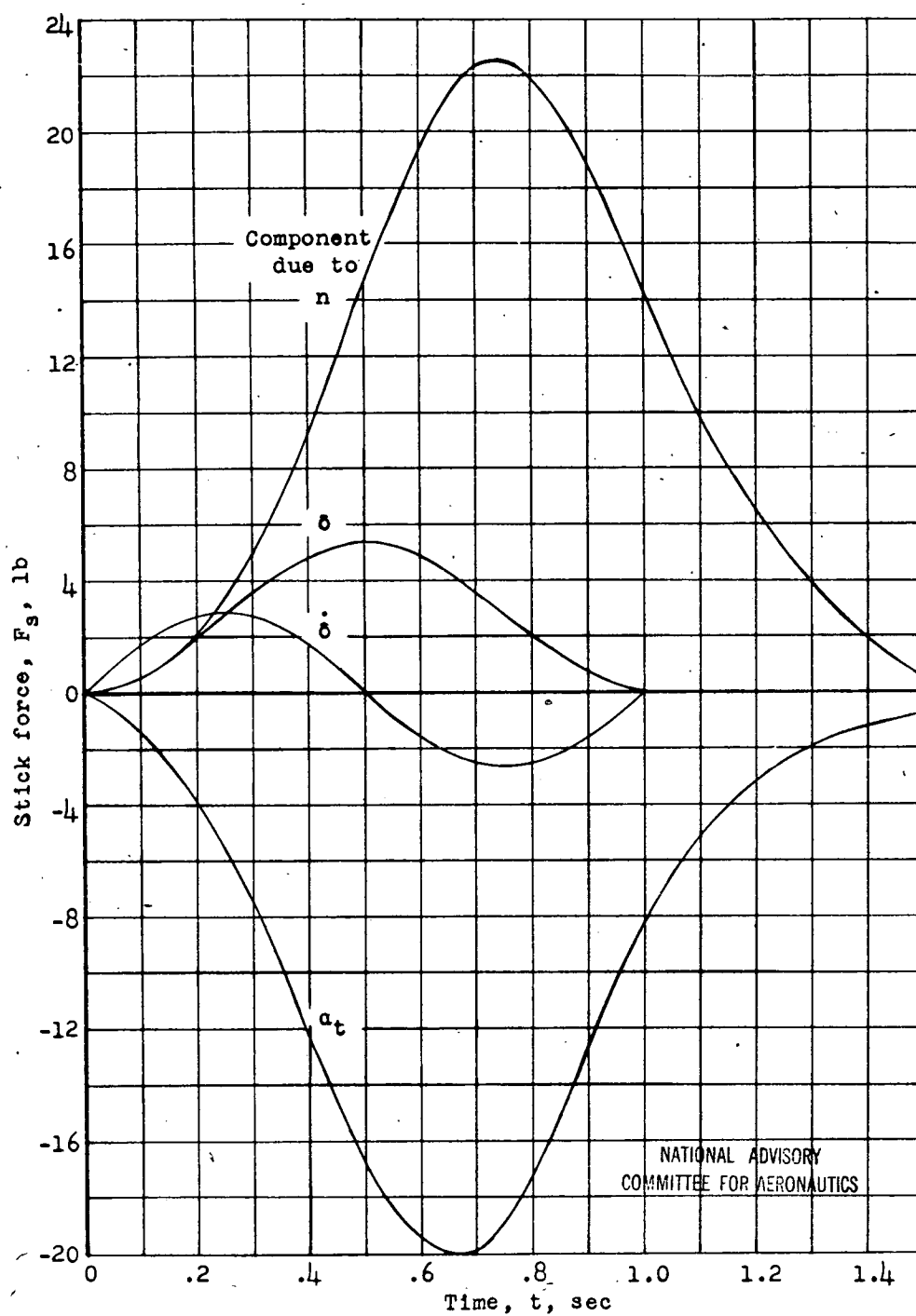


Figure 8.- Stick force and normal acceleration due to rapid elevator motion.  $T = 2$  seconds;  $V = 400$  miles per hour;  $x_{a.c.} = 0.01c$ .

Figure 9.- Components of stick force for case  $F_4$  in figure 5.

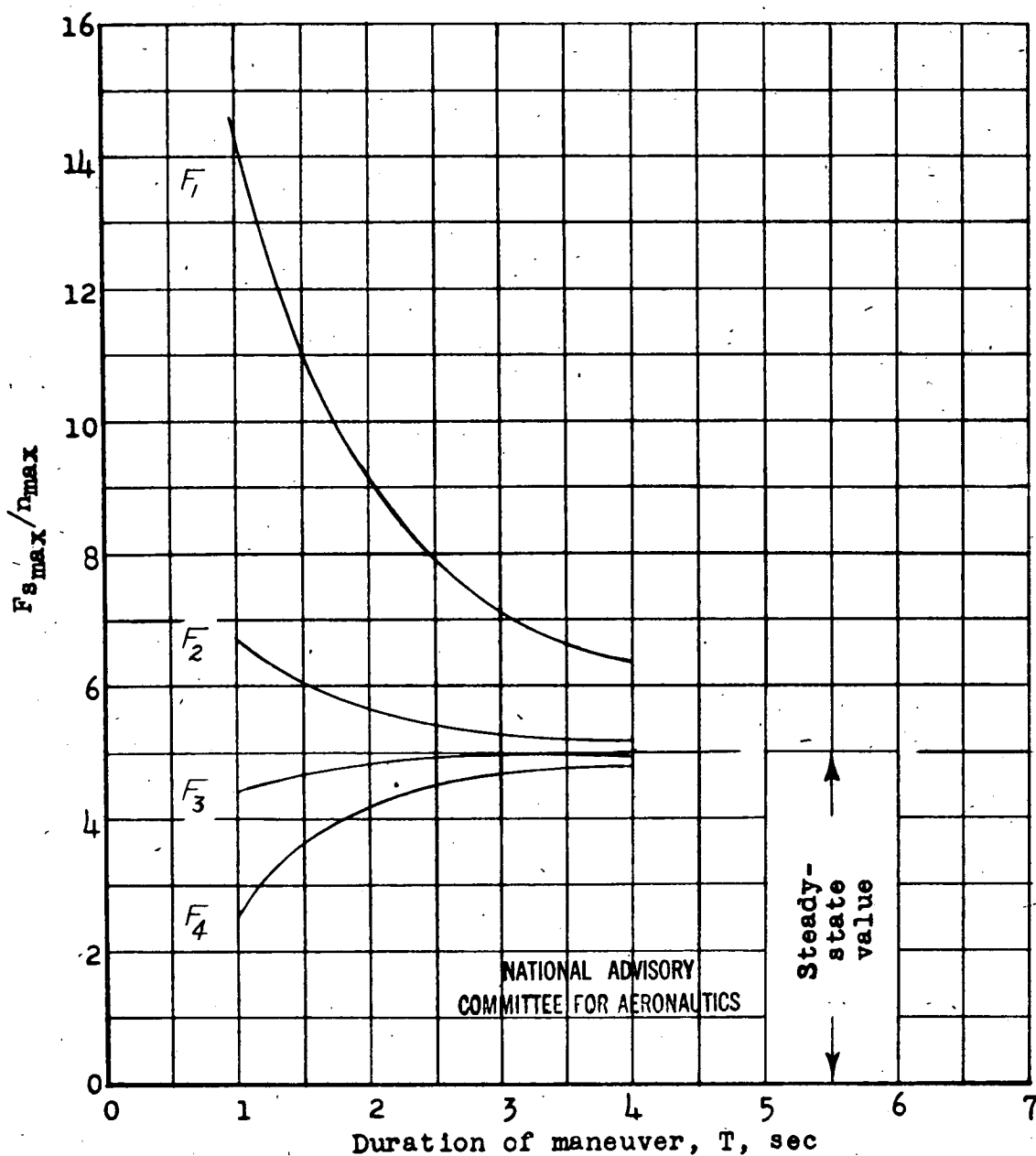


Figure 10.- Maximum stick force per unit maximum acceleration against duration of maneuver.  $x_{a.c.} = 0.075c$ .

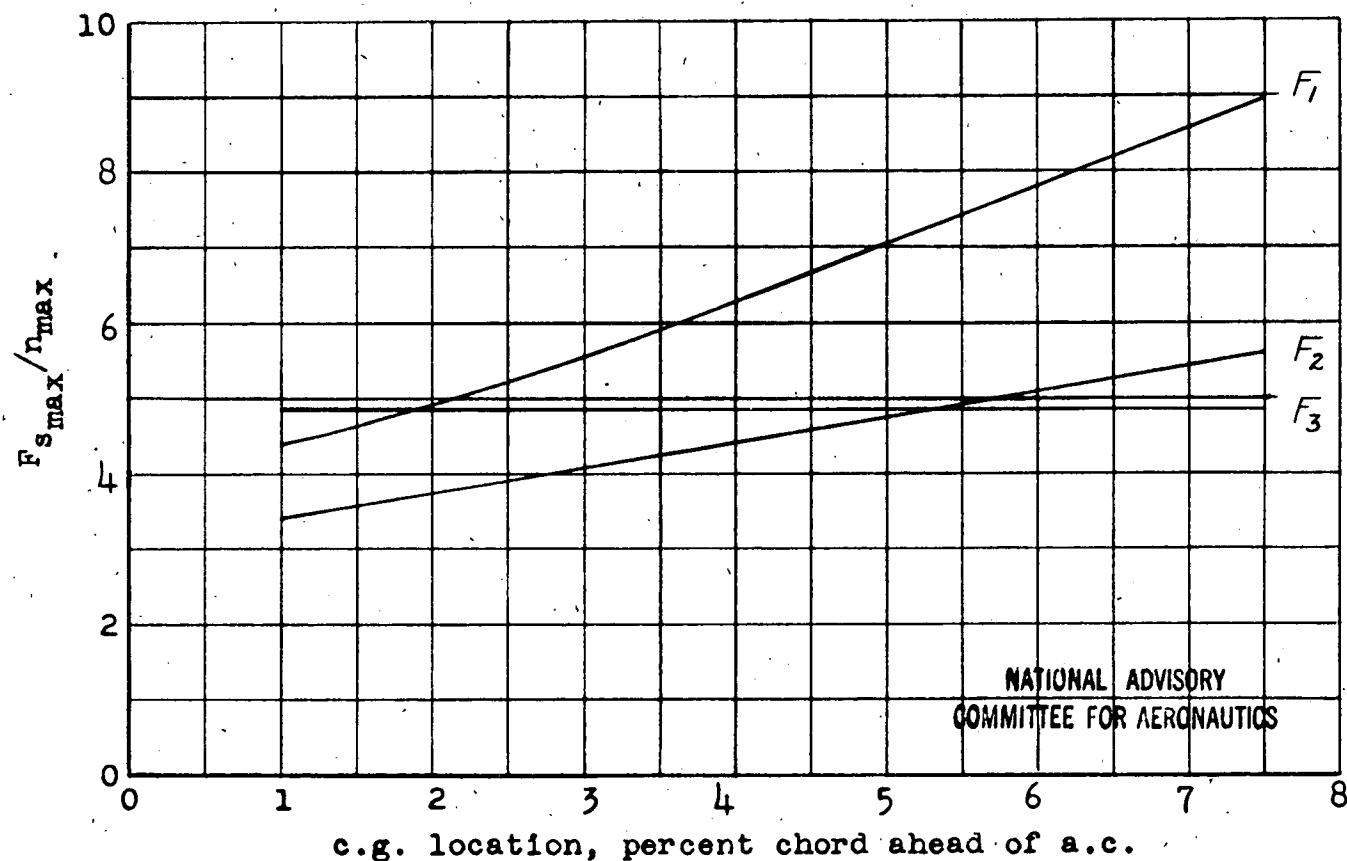


Figure 11.- Variation of maximum stick force per unit maximum acceleration with center-of-gravity location.  $T = 2$  seconds;  $V = 400$  miles per hour.